2.1 LINE-TO-GROUND FAULT

Figure 2-1(a) shows that phase a of a three-phase system goes to ground through an impedance Z_f. The flow of ground fault current depends on the method of system grounding. A solidly grounded system with zero ground resistance is assumed. There will be some impedance to flow of fault current in the form of impedance of the return ground conductor or the grounding grid resistance. A ground resistance can be added in series with the fault impedance Z_f. The ground fault current must have a return path through the grounded neutrals of generators or transformers. If there is no return path for the ground current, $Z_0 = \infty$ and the ground fault current is zero. This is an obvious conclusion.

Phase a is faulted in Fig. 2-1(a). As the load current is neglected, currents in phases b and c are zero, and the voltage at the fault point, $V_a = I_a Z_f$. The sequence components of the currents are given by

$$\begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} = \frac{1}{3}\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}\begin{vmatrix} I_a \\ 0 \\ 0 \end{vmatrix} = \frac{1}{3}\begin{vmatrix} I_a \\ I_a \\ I_a \end{vmatrix} \tag{2.1}$$

Also,

$$I_0 = I_1 = I_2 = \frac{1}{3} I_a \tag{2.2}$$

$$3I_0 Z_f = V_0 + V_1 + V_2 = -I_0 Z_0 + (V_a - I_1 Z_1) - I_2 Z_2 \tag{2.3}$$

which gives

$$I_0 = \frac{V_a}{Z_0 + Z_1 + Z_2 + 3Z_f} \tag{2.4}$$

The fault current I_a is

$$I_a = 3I_0 = \frac{3V_a}{(Z_1 + Z_2 + Z_0) + 3Z_f} \tag{2.5}$$

This shows that the equivalent fault circuit using sequence impedances can be constructed as shown in Fig. 2-1(b). In terms of sequence impedances' network blocks the connections are shown in Fig. 2-1(c).

This result could also have been arrived at from Fig. 2-1(b):

$$(V_a - I_1 Z_1) + (-I_2 Z_2) + (-I_0 Z_0) - 3Z_f I_0 = 0$$

which gives the same equations (2.4) and (2.5). The voltage of phase b to ground under fault conditions is

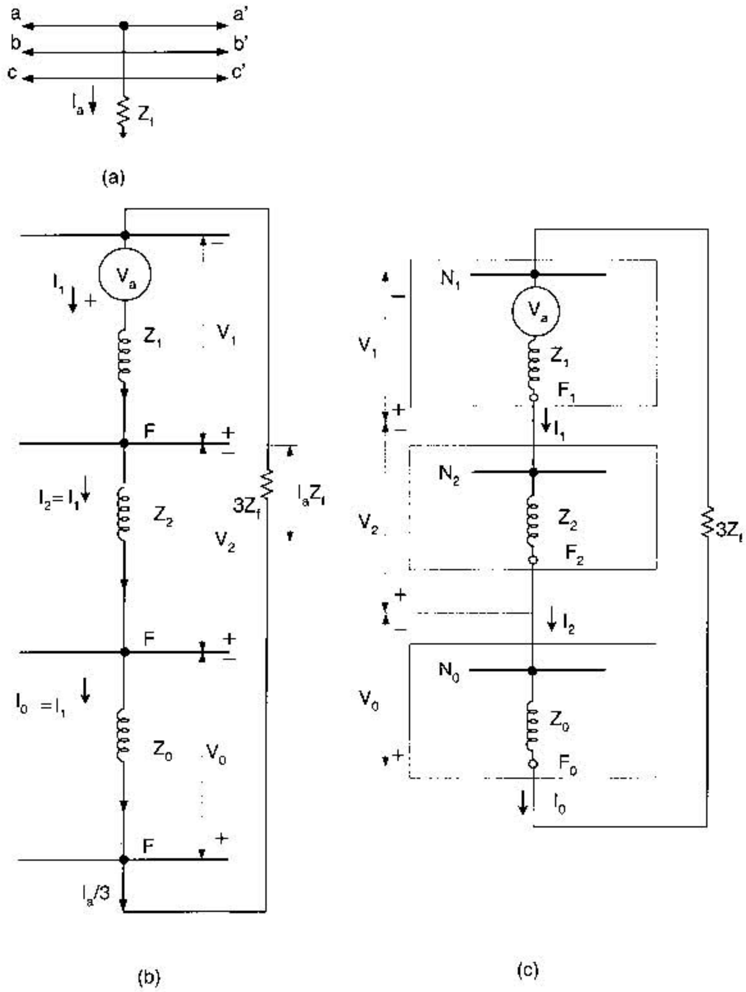

Figure 2-1 (a) Line-to-ground fault in a three-phase system; (b) line-to-ground fault equivalent circuit; (c) sequence network interconnections.

$$V_b = a^2 V_1 + aV_2 + V_0$$
$$= V_a \frac{3a^2 Z_f + Z_2(a^2 - a) + Z_0(a^2 - 1)}{(Z_1 + Z_2 + Z_0) + 3Z_f} \tag{2.6}$$

Similarly, the voltage of phase c can be calculated.

An expression for the ground fault current for use in grounding grid designs and system grounding is as follows:

$$I_a = \frac{3V_a}{(R_0 + R_1 + R_2 + 3R_f + 3R_G) + j(X_0 + X_1 + X_2)} \tag{2.7}$$

where R_f is the fault resistance and R_G is the resistance of the grounding grid; R_0, R_1, and R_2 are the sequence resistances and X_0, X_1, and X_2 are sequence reactances.

2.2 LINE-TO-LINE FAULT

Figure 2-2(a) shows a line-to-line fault. A short-circuit occurs between phases b and c, through a fault impedance Z_f. The fault current circulates between phases b and c, flowing back to source through phase b and returning through phase c; $I_a = 0$, $I_b = -I_c$. The sequence components of the currents are

$$\begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} = \frac{1}{3}\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}\begin{vmatrix} 0 \\ -I_c \\ I_c \end{vmatrix} = \frac{1}{3}\begin{vmatrix} 0 \\ -a + a^2 \\ -a^2 + a \end{vmatrix} \tag{2.8}$$

From Eq. (2.8), $I_0 = 0$ and $I_1 = -I_2$.

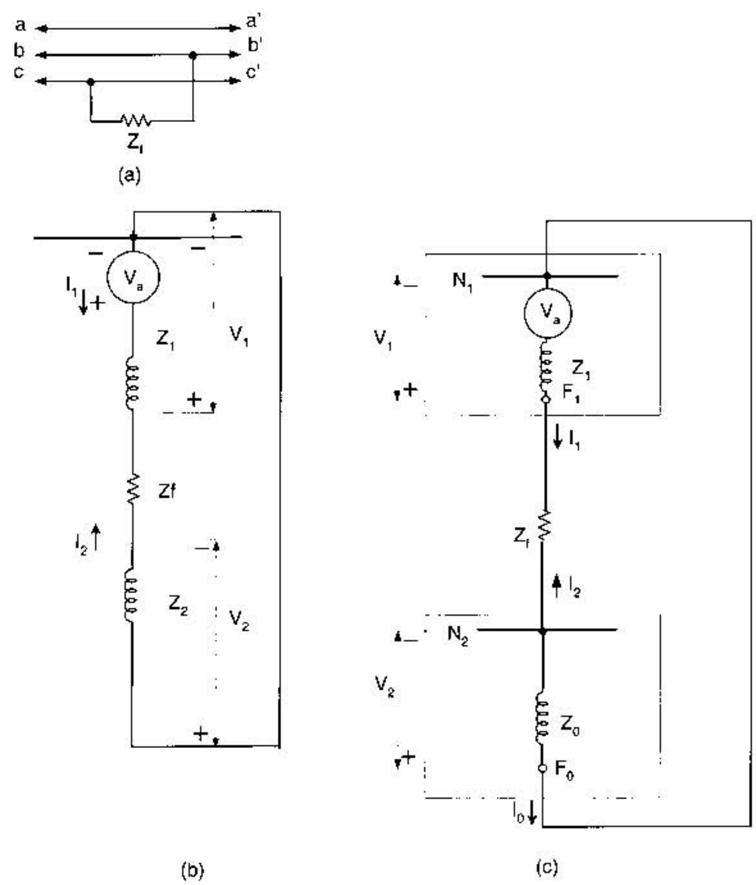

Figure 2-2 (a) Line-to-line fault in a three-phase system; (b) line-to-line fault equivalent circuit; (c) sequence network interconnections.

$$V_b - V_c = \begin{vmatrix} 0 & 1 & -1 \end{vmatrix} \begin{vmatrix} V_a \\ V_b \\ V_c \end{vmatrix} = \begin{vmatrix} 0 & 1 & -1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} V_0 \\ V_1 \\ V_2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a^2 - a & a - a^2 \end{vmatrix} \begin{vmatrix} V_0 \\ V_1 \\ V_2 \end{vmatrix} \tag{2.9}$$

Therefore,

$$\begin{aligned} V_b - V_c &= (a^2 - a)(V_1 - V_2) \\ &= (a^2 I_1 + a I_2) Z_f \\ &= (a^2 - a) I_1 Z_f \end{aligned} \tag{2.10}$$

This gives

$$(V_1 - V_2) = I_1 Z_f \tag{2.11}$$

The equivalent circuit is shown in Fig. 2-2(b) and (c).

Also

$$I_b = (a^2 - a) I_1 = -j\sqrt{3} I_1 \tag{2.12}$$

and,

$$I_1 = \frac{V_a}{Z_1 + Z_2 + Z_f} \tag{2.13}$$

The fault current is

$$I_b = -I_c = \frac{-j\sqrt{3} V_a}{Z_1 + Z_2 + Z_f} \tag{2.14}$$

2.3 DOUBLE LINE-TO-GROUND FAULT

A double line-to-ground fault is shown in Fig. 2-3(a). Phases b and c go to ground through a fault impedance Z_f. The current in the ungrounded phase is zero, i.e., $I_a = 0$. Therefore, $I_1 + I_2 + I_0 = 0$.

$$V_b = V_c = (I_b + I_c) Z_f \tag{2.15}$$

Thus,

$$\begin{vmatrix} V_0 \\ V_1 \\ V_2 \end{vmatrix} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} V_a \\ V_b \\ V_b \end{vmatrix} = \frac{1}{3} \begin{vmatrix} V_a + 2V_b \\ V_a + (a + a^2) V_b \\ V_a + (a + a^2) V_b \end{vmatrix} \tag{2.16}$$

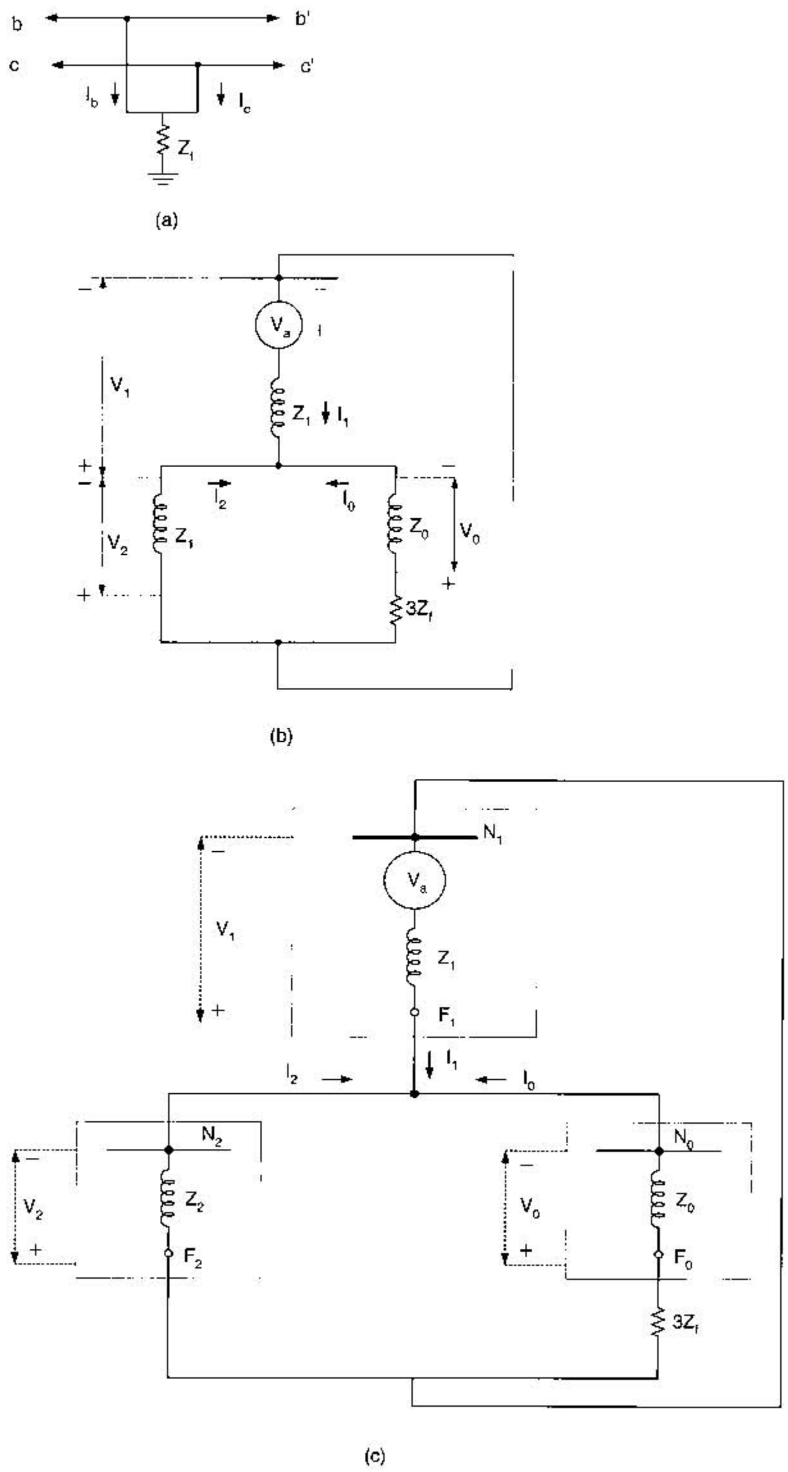

Figure 2-3 (a) Double line-to-ground fault in a three-phase system; (b) double line-to-ground fault equivalent circuit; (c) sequence network interconnections.

which gives $V_1 = V_2$ and

$$
\begin{aligned}
V_0 &= \frac{1}{3}(V_a + 2V_b) \\
&= \frac{1}{3}[(V_0 + V_1 + V_2) + 2(I_b + I_c)Z_f] \\
&= \frac{1}{3}[(V_0 + 2V_1) + 2(3I_0)Z_f] \\
&= V_1 + 3Z_f I_0
\end{aligned}
\tag{2.17}
$$

This gives the equivalent circuit of Fig. 2-3(b) and (c).

The fault current is

$$
\begin{aligned}
I_1 &= \frac{V_a}{Z_1 + [Z_2 \| (Z_0 + 3Z_f)]} \\
&= \frac{V_a}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}}
\end{aligned}
\tag{2.18}
$$

2.4 THREE-PHASE FAULT

The three phases are short-circuited through equal fault impedances Z_f, Fig. 2-4(a). The vectorial sum of fault currents is zero, as a symmetrical fault is considered and there is no path to ground.

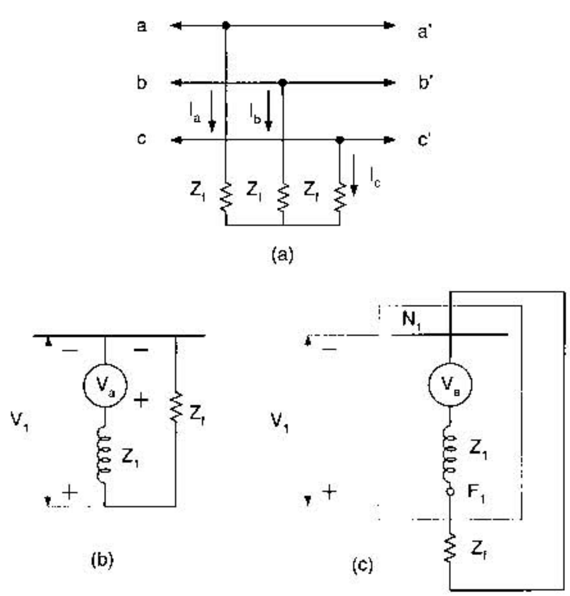

Figure 2-4 (a) Three-phase symmetrical fault; (b) equivalent circuit; (c) sequence network.

$$I_0 = 0 \; I_a + I_b + I_c = 0 \tag{2.19}$$

As the fault is symmetrical:

$$\begin{vmatrix} V_a \\ V_b \\ V_c \end{vmatrix} = \begin{vmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{vmatrix} \begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix} \tag{2.20}$$

The sequence voltages are given by

$$\begin{vmatrix} V_0 \\ V_1 \\ V_2 \end{vmatrix} = [T_s]^{-1} \begin{vmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{vmatrix} [T_s] \begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{vmatrix} \begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} \tag{2.21}$$

This gives the equivalent circuit of Fig. 2-4(b) and (c).

$$\begin{aligned} I_a &= I_1 = \frac{V_a}{Z_1 + Z_f} \\ I_b &= a^2 I_1 \\ I_c &= a I_1 \end{aligned} \tag{2.22}$$

2.8.1 Two-Conductor Open Fault

Consider that conductors of phases b and c are open-circuited. The currents in these conductors then go to zero.

$$I_b = I_c = 0 \tag{2.23}$$

The voltage across the unbroken phase conductor is zero, at the point of break, Fig. 2-13(a).

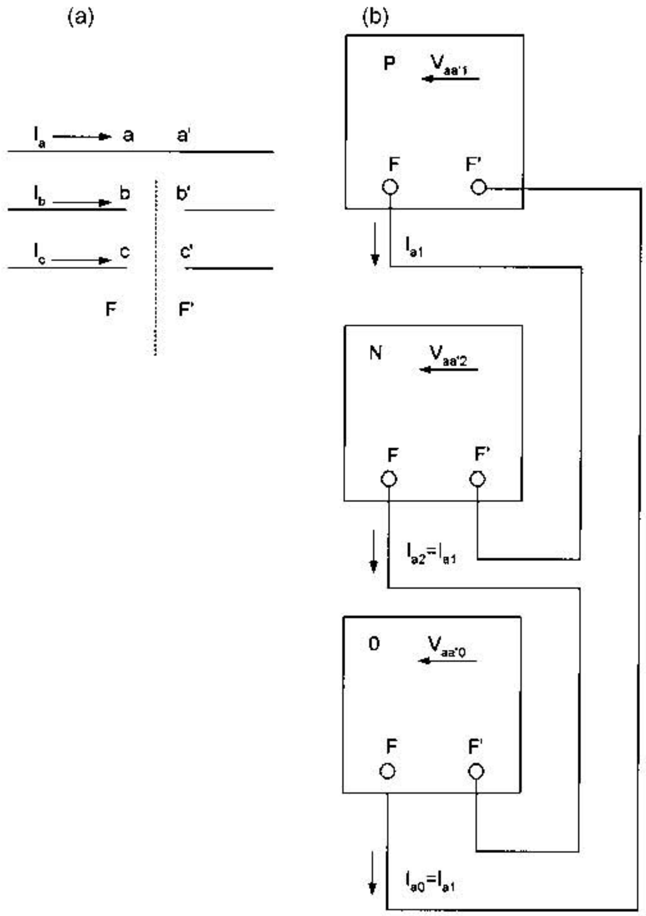

Figure 2-13 (a) Two-conductor open series fault; (b) connection of sequence networks.

$$
\begin{aligned}
&V_{a0} = V_{ao1} + V_{ao2} + V_{a0} = 0 \\
&I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a
\end{aligned}
\tag{2.24}
$$

This suggests that sequence networks can be connected in series as shown in Fig. 2-13(b).

2.8.2 One Conductor Open

Now consider that phase a conductor is broken, Fig. 2-14(a)

$$I_a = 0 \qquad V_{b0} - V_{c0} = 0 \tag{2.25}$$

Thus,

$$
\begin{aligned}
&V_{ao1} = V_{ao2} = V_{ao0} = \frac{1}{3} V_{ao} \\
&I_{ai} + I_{a2} + I_{a0} = 0
\end{aligned}
\tag{2.26}
$$

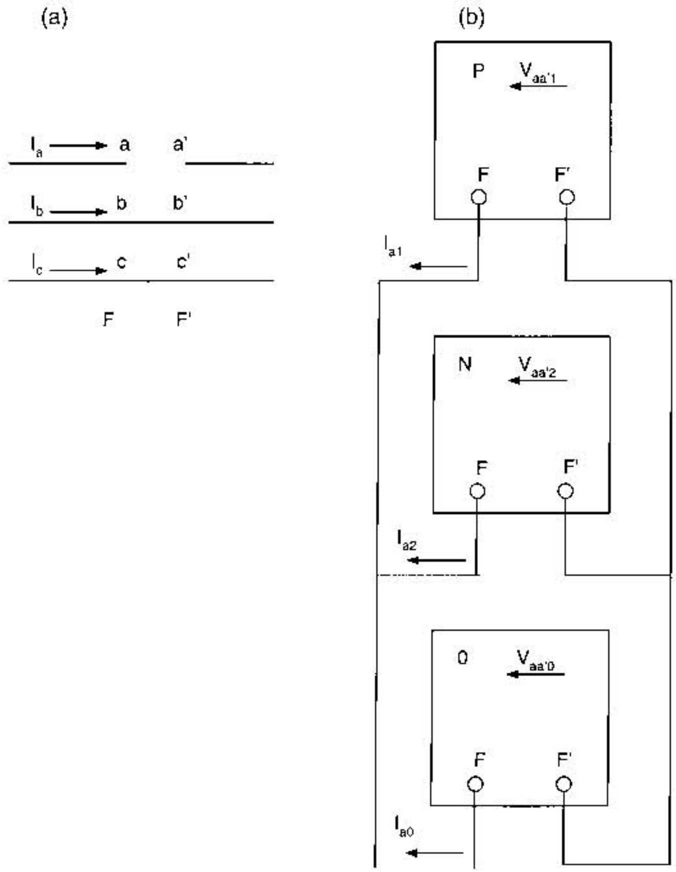

Figure 2-14 (a) One-conductor open series fault; (b) connection of sequence networks.